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BRIEF COMMUNICATION

A UNIFIED THEORY ON PARTICLE TRANSPORT IN A TURBULENT DILUTE TWO-PHASE SUSPENSION FLOW—II

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INTRODUCTION

The dynamics of a turbulent two-phase suspension flow plays a fundamental role in the study of a large number of technical problems of great importance. Most earlier conventional theoretical treatments, based on the flow-field classifications of single-phase flows, divide the flow field into two fixed transverse regions: the turbulent diffusion controlled core region and the mean fluid motion controlled quasi-laminar region (e.g. Lin *et al.* 1953; Friedlander & Johnstone 1951; Davies 1966; Beal 1968; Hutchinson *et al.* 1971). In the turbulent diffusion controlled core region, the particle motion is assumed to be controlled entirely by turbulent fluid oscillations and the particles are assumed to be transported in the same way as scalar quantities by the turbulent diffusion of the fluid. In the mean fluid motion controlled quasi-laminar region, the particle motion is assumed to be controlled entirely by the mean fluid motion through the drag interaction. Results of calculations using this scheme on the particulate wall deposition have been found to differ from measurements by as much as 4 orders of magnitude (Wildi 1982). There seems to be a serious question on the correctness of the very physics assumed in the scheme.

Specifically, there are three areas of weakness in the aforementioned scheme. First, in the turbulent core, the particles are assumed to be transported entirely by the turbulent diffusion of the fluid. Secondly, in the quasi-laminar region, the particle transport is assumed to be controlled entirely by the viscous drag. Thirdly, the location of the divide between these two transverse regions is fixed and is to be determined from the flow-field classification of the corresponding single-phase flow, combining the turbulent core and the buffer zone to form the turbulent core for the suspension flow and identifying the viscous sublayer as the quasi-laminar region for the suspension flow.

Based on a concept of particle frequency response in an oscillating flow field first developed by Hjelmfelt & Mockros (1966), Rouhiainen & Stachiewicz (1970) pointed out that in the turbulent core the validity of the turbulent diffusion assumption for particle transport can be characterized by the value of an amplitude ratio, η, which is defined as the ratio of the amplitude of the particle oscillation to the surrounding eddy motion. They also observed that in the quasi-laminar region the particle transport cannot be controlled only by the viscous drag, since the effect of the shear-flow-induced transverse lift force, first derived by Saffman (1965), is no longer negligible. The importance of including this lift force has already been recognized in the analyses of the simpler laminar boundary-layer flows of a two-phase suspension by Otterman & Lee (1969, 1970). Their theoretical treatment has been verified experimentally by the use of laser–Doppler anemometry by Lee & Einav (1972).

Lee & Durst (1979, 1980, 1982) used the concept of particle frequency response in an oscillating flow field to examine the question of the location of the divide between the two transverse regions of the turbulent core and the quasi-laminar region. Based on the known turbulence properties of corresponding single-phase flows, their analysis gives the location of the divide, or the transverse cutoff location, as a function of particle size, physical system properties and flow properties. Some of the conclusions of their theory, with the incorporation of the shear-flow-induced transverse lift force, have shown only qualitative improvement over the previous theories in comparison with local velocity measurements by laser–Doppler anemometry in particulate two-phase pipe flows by Lee & Durst (1982).

The main shortcoming of these theories lies in the assumption of an artificial boundary which separates the two artificial transverse regions: the turbulent core and the quasi-laminar region. They deal with these two regions as two separate entities. The accompanying analyses usually contain two unconnected sets of governing equations, one for each of the two regions, having no physical or mathematical ties with each other. Therefore, these theories cannot produce a smooth transition of flow properties across the flow field in the transverse direction. The physics of such a flow, however, call for such a smooth transition of these flow properties. The turbulent diffusion is expected to continuously contribute, generally in decreasing extent and importance, to the transverse particle transport, from the turbulent core to the quasi-laminar region next to the boundary wall. In parallel, the quasi-laminar interaction based on the mean flow properties of the two phases is expected to contribute, generally in decreasing extent and importance, to the transverse particle transport, from the quasi-laminar region next to the boundary wall to the turbulent core.

Other weaknesses in these theories include the lack of a rational explanation of the turbulent particle diffusion mechanism assumed to exist and the unjustified use of known flow properties of corresponding single-phase flows in the study of particle transport in turbulent two-phase suspension flows. Therefore, a new unified theoretical approach for the entire transverse flow region from the turbulent core to the quasi-laminar region next to the boundary wall is much in order. In this approach, the turbulent particle diffusion mechanism should be formulated from a rational basis and the flow properties of the fluid needed in the analysis should be those obtained from realistic turbulent two-phase suspension flows.

ANALYSES

Lee & Wiesler (1987) have developed a new theoretical model to explain the behavior of transverse particle transport in turbulent flow of a dilute two-phase suspension due to turbulent diffusion alone. This model is based on the ability of a particle to respond to surrounding fluid motion, and depends on particle size and density relative to the carrier fluid, the fractional variation in particle concentration in the transverse direction as well as the existing turbulence structure of the surrounding fluid. The fluid velocity, as seen by a particular particle in the suspension, is divided into two superimposed components representing, respectively, turbulent fluid fluctuations and an apparent local fluid drifting velocity due to the effect on the oscillatory component of fluid motion by the transverse concentration distribution of particles. The apparent local fluid drifting velocity has been found to be a function of the turbulence properties of the surrounding fluid and the fractional particle concentration gradient in the transverse direction. However, there are two inherent drawbacks in this approach. First, the turbulence properties of the surrounding fluid used are those of a corresponding single-phase flow instead of those of the realistic two-phase suspension flow. Secondly, this approach has made use of the equation of motion of a spherical particle in a turbulent fluid flow, first formulated by Tchen (1947) and reviewed by Hinze (1959) and subsequently rederived from first principles by Maxey & Riley (1983). The assumed model for the derivation of this equation is a single spherical particle being subjected to the laminar viscous interaction of an infinite, locally laminar ambient fluid motion at a small particle Reynolds number based on the molecular viscosity of the fluid. In an actual turbulent two-phase suspension flow, a particle sees a turbulent instead of a laminar ambient flow and, since it is not alone in the flow, the inherent turbulence structure of its surrounding ambient flow can be conceivably modified significantly, due to the presence of other particles. Furthermore, for large particles, especially under the influence of gravity, the particle Reynolds number may in effect be very large.

An analysis by Lee (1987) of two recent measurements of upward turbulent flows of a solid particle–air two-phase dilute suspension in a vertical pipe by laser–Doppler anemometry (Lee & Durst 1982; Tsuji *et al.* 1984) leads to the determination of the realistic particle drag coefficient over a wide range of physical and flow conditions. It is established that the drag on a particle in a turbulent two-phase suspension flow can be described, for extended values of the particle

Reynolds number, by a simple pseudo Stokes law based on an apparent turbulent viscosity of the fluid for the particles in the suspension flow. A correlation is provided for this apparent turbulent viscosity in terms of the particle size and concentration in the suspension, the local flow turbulence Reynolds number and the particle/fluid density ratio.

When a spherical particle moves faster or lags behind the fluid in a laminar ambient shear flow, it experiences a lift force in the transverse direction together with a drag force in the longitudinal direction. For small values of the particle Reynolds number in a laminar flow, Saffman (1965) formulated an expression for this shear-slip-induced lift force, in addition to an expression for the drag force which is identical to the classical Stokes drag derived for a particle moving in a uniform laminar ambient fluid flow at small values of the particle Reynolds number. Rubin (1977) performed experiments to measure both the transverse lift force and the longitudinal drag force, at small values of the particle Reynolds number, on a sphere subject to ambient flow with locally uniform shear in a water channel. In all cases, with the allowance of the scatter of the data, Saffman's (1965) theoretical predictions were essentially confirmed. The most surprising thing is that while Saffman's formulation was based on an unbounded ambient fluid flow, this confirmation has always held, including the worst possible case in which the sphere is practically touching the wall.

Lee's (1987) analysis of realistic turbulent two-phase suspension flows establishes that the drag on a particle in the suspension can be described, for extended values of the particle Reynolds number, by the same simple Stokes law, which governs the drag on a particle in a laminar ambient fluid flow, with the molecular viscosity of the fluid replaced by an apparent turbulent viscosity of the fluid felt by the particles in the turbulent suspension flow. A by-product of this analysis is the realization of relaxation of the restriction to small values of the particle Reynolds number on the formulation of the transverse lift force on the particle in extending the study of the motion of a particle in a laminar fluid flow to the motion of a particle in a turbulent two-phase suspension flow. Therefore, the transverse lift force on a particle in a turbulent two-phase suspension flow can be described, for extended values of the particle Reynolds number, by a simple pseudo Saffman law based on an apparent turbulent viscosity of the fluid for the particles in the suspension flow.

GOVERNING EQUATIONS

The apparent drifting velocity of the fluid, due to the interaction between the transverse fractional concentration gradient of particles and the turbulent fluctuations of the fluid, derived under simplifying assumptions by Lee & Wiesler (1987) for the restricted case of particle transport by simple turbulent diffusion will now be modified and extended to the general case of particle transport in which turbulent diffusion plays only a part. The modifications comprise the replacement of the molecular viscosity of the fluid by the apparent turbulent viscosity of the fluid felt by the particles in the suspension flow and the substitution of flow properties obtained from corresponding single-phase flows by flow properties obtained from realistic two-phase suspension flows. With these modifications, the apparent drifting velocity of the fluid which is felt by a particle in a turbulent two-phase suspension flow becomes

$$\bar{\bar{v}}_{fd} = -\frac{2\pi}{\omega_e}\bar{\eta}_e^2\overline{(\tilde{v}_{fs})^2}\frac{1}{\alpha}\frac{\partial \alpha}{\partial y}, \qquad [1]$$

where

$\bar{\bar{v}}_{fd}$ = the apparent drifting velocity of the fluid which is felt by a particle in a turbulent two-phase suspension flow,

ω_e = frequency of oscillation associated with the average energy across the turbulence spectrum of the ambient fluid motion in the suspension flow

$\bar{\eta}_e = \{[1 + f_1(\omega_e)]^2 + [f_2(\omega_e)]^2\}^{\frac{1}{2}}$, the particle/fluid oscillation amplitude ratio corresponding to the oscillation frequency in a turbulent two-phase suspension flow,

$$f_1(\omega_e)=\frac{\omega_e\left[\omega_e+\tilde{c}\left(\frac{\pi\omega_e}{2}\right)^{\frac{1}{2}}\right](b-1)}{\left[\tilde{a}+\tilde{c}\left(\frac{\pi\omega_e}{2}\right)^{\frac{1}{2}}\right]^2+\left[\omega_e+\tilde{c}\left(\frac{\pi\omega_e}{2}\right)^{\frac{1}{2}}\right]^2},$$

$$f_2(\omega_e)=\frac{\omega_e\left[\tilde{a}+\tilde{c}\left(\frac{\pi\omega_e}{2}\right)^{\frac{1}{2}}\right](b-1)}{\left[\tilde{a}+\tilde{c}\left(\frac{\pi\omega_e}{2}\right)^{\frac{1}{2}}\right]^2+\left[\omega_e+\tilde{c}\left(\frac{\pi\omega_e}{2}\right)^{\frac{1}{2}}\right]^2},$$

$$\tilde{a}=\frac{36\tilde{\mu}_f}{(2\rho_p+\rho_f)d_p^2},$$

$$b=\frac{3\rho_f}{(2\rho_p+\rho_f)},$$

$$\tilde{c}=18\frac{\left(\frac{\rho_f\tilde{\mu}_f}{\pi}\right)^{\frac{1}{2}}}{(2\rho_p+\rho_f)d_p},$$

$\tilde{\mu}_f$ = the apparent turbulent viscosity of the fluid which is felt by a particle in a turbulent two-phase suspension flow, provided by the correlation of Lee (1987),

ρ_p, ρ_f = the intrinsic densities of the particles and the fluid, respectively,

$\overline{(\tilde{v}_{fs})^2}$ = the square of turbulence intensity in the transverse direction of fluid in a turbulent two-phase suspension flow,

α = the local time-mean particle volumetric concentration,

y = the transverse coordinate,

and

d_p = the particle diameter.

An analysis by Lee & Börner (1987) of two recent measurements of upward turbulent flows of a solid particle–air two-phase dilute suspension by laser–Doppler anemometry (Lee & Durst 1982; Tsuji *et al.* 1984) leads to the determination of the realistic flow turbulence properties over a wide range of physical and flow conditions. In particular, correlations of realistic values of $\overline{(\tilde{v}_{fs})^2}$ and ω_e are provided in terms of the particle size and concentration in the suspension, the local flow turbulence Reynolds number and the particle/fluid density ratio.

The apparent time-mean fluid velocity in the transverse direction which is felt by a particle in the suspension is then

$$\bar{v}_f=v_{f0}+\bar{v}_{fd}=v_{f0}-\frac{2\pi}{\omega_e}\eta_e^2\overline{(\tilde{v}_{fs})^2}\frac{1}{\alpha}\frac{\partial\alpha}{\partial y}, \qquad [2]$$

where v_{f0} = the time-mean fluid velocity in the transverse direction ($v_{f0}=0$ for flow in a straight pipe).

Taking into consideration the above discussions, we can now formulate the following governing equations for the time-mean motion of particles in a turbulent flow of a two-phase suspension:

in the longitudinal (x) *direction*

$$\frac{d\bar{u}_p}{dt}=\tilde{a}(u_{f0}-\bar{u}_p)+b'a_x; \qquad [3]$$

in the transverse (y) *direction*

$$\frac{d\bar{v}_p}{dt}=\tilde{a}\left\{\left[v_{f0}-\frac{2\pi}{\omega_e}\eta_e^2\overline{(\tilde{v}_{fs})^2}\frac{1}{\alpha}\frac{\partial\alpha}{\partial y}\right]-\bar{v}_p\right\}+\tilde{h}\left|\frac{\partial u_{f0}}{\partial y}\right|^{\frac{1}{2}}(\bar{u}_p-u_{f0})+b'a_y; \qquad [4]$$

where

$\bar{u}_p, \bar{v}_p$ = the time-mean particle velocities in the longitudinal and transverse directions, respectively,

u_{f0} = the time-mean fluid velocity in the longitudinal direction, provided by the correlation of Lee & Börner (1987) in the case of upward flow of a suspension in a vertical pipe,

a_x, a_y = components of acceleration due to gravitational body force in the longitudinal (x) and transverse (y) directions, respectively (in the case of upward flows through a vertical pipe, $a_y = 0$ and $a_x = -g$, where g is the gravitational acceleration),

x = the longitudinal coordinate,

$$b' = 2\frac{(\rho_p - \rho_f)}{(2\rho_p + \rho_f)}$$

and

$$\tilde{h} = \frac{6.17(\tilde{u}_f\rho_f)^{\frac{1}{2}}}{(2\rho_p + \rho_f)d_p}.$$

The first terms on the r.h.s. of [3] and [4] stand for the pseudo Stokes drag force terms in the longitudinal and transverse directions, respectively, which are now valid for extended values of the particle Reynolds number. The second term on the r.h.s. of [4] stands for the term for the pseudo Saffman lift force, which is now valid for extended values of the particle Reynolds number. The last terms on the r.h.s. of [3] and [4] stand for the components of the combination of the weight and the buoyancy force on a particle.

The instantaneous velocity of a particle in the transverse direction is the sum of its time-mean velocity in the transverse direction, $\bar{v}_p$, and its time-dependent fluctuating velocity in the transverse direction, $\tilde{v}_{ps}$:

$$v_p = \bar{v}_p + \tilde{v}_{ps}. \qquad [5]$$

Lee & Wiesler (1987) also obtained theoretically the amplitude of the particle fluctuating velocity based on the molecular viscosity of the fluid and flow turbulence properties from corresponding single-phase flows. Replacing the molecular viscosity of the fluid by the apparent turbulent viscosity of the fluid felt by a particle in the suspension flow and the flow turbulence properties from corresponding single-phase flows by those from realistic turbulent two-phase suspension flows, we have the amplitude of the particle fluctuating velocity $\tilde{v}_{ps}$:

$$[\overline{(\tilde{v}_{ps})^2}]^{\frac{1}{2}} = \tilde{\eta}_e[\overline{(\tilde{v}_{fs})^2}]^{\frac{1}{2}}. \qquad [6]$$

In this derivation, it is assumed that the particle transverse fluctuation velocity $\tilde{v}_{ps}$ is oscillating at the same frequency as the frequency of oscillation ω_e associated with the average energy across the turbulence spectrum of the ambient fluid motion in the suspension flow.

DISCUSSION

The instantaneous transverse particle velocity v_p is the superposition of the time-mean transverse particle velocity v_p and the transverse particle fluctuating velocity $\tilde{v}_{ps}$. When a particle moves towards a boundary wall, the dominant frequency of oscillation ω_e will increase and the amplitude of oscillation of the ambient fluid motion will decrease (Lee & Börner 1987). Since the amplitude ratio $\tilde{\eta}_e$ will decrease with an increase in ω_e, the amplitude of the transverse fluctuating velocity of the particle will gradually decrease and eventually vanish on moving towards the wall, according to [6].

The time-mean motion of a particle in a suspension flow is governed by [3] and [4]. The longitudinal time-mean particle velocity $\bar{u}_p$ is linked to the transverse time-mean particle velocity $\bar{v}_p$ through the presence of $\bar{u}_p$ in the pseudo Saffman transverse lift force in [4], the governing equation for $\bar{v}_p$. In the transverse direction of decreasing time-mean longitudinal time-mean fluid velocity u_{f0}, the pseudo Saffman transverse lift force will work either against or with the pseudo Stokes drag force depending on whether the particle lags or leads the fluid in the longitudinal direction.

Lee & Durst (1982) measured the turbulent flow of air, carrying single-sized glass spheres in suspension, upwards along a vertical pipe by laser–Doppler anemometry. Mean vertical velocity profiles were obtained both for the air and the particles. Particle velocities were more uniformly distributed over the section than the air velocity and were generally lower. However, for particles of 100 μm dia, particle velocities exceeded the local air velocity over a region extending from the wall for about 20% of the pipe radius. For particles of 200 μm dia, this region extended for about 10% of the pipe radius; for larger particles this effect was not obtained. On the other hand, there was a region extending from the wall which was free of particles, extending for about 5% of the pipe radius in the case of 400 μm dia particles and for about 10% for 800 μm dia particles. Now, an attempt will be made to give a physical explanation to these peculiar measured phenomena from the present particle transport theory with the use of the apparent turbulent viscosity of the fluid, which is felt by a particle in the suspension, provided by the correlation of Lee (1987), and the turbulent flow properties of air in a two-phase suspension flow, provided by the correlations of Lee & Börner (1987).

In the flow through a vertical straight pipe, in which $v_{f0} = 0$, $a_x = -g$ and $a_y = 0$, the transverse transport of particles is controlled jointly by the pseudo Stokes drag force due to the difference between the time-mean transverse velocity of the fluid, which comprises now only the drifting velocity of the fluid, the time-mean transverse velocity of the particle and the pseudo Saffman lift force. For large particles, e.g. the particles of 400 and 800 μm dia in Lee & Durst's (1982) experiments, the transverse particle transport in the central portion of the pipe is insignificant where the fluid drifting velocity is small, due to the smallness of $\tilde{\eta}_e$ and

$$\left|\frac{1}{\alpha}\frac{\partial \alpha}{\partial y}\right|,$$

and the pseudo Saffman lift force is small due to the smallness of $|\partial u_{f0}/\partial y|$. In the near-wall region, the fluid drifting velocity is increased somewhat due to an increase in

$$\left|\frac{1}{\alpha}\frac{\partial \alpha}{\partial y}\right|,$$

but still remains small due to the continued smallness of $\tilde{\eta}_e$. However, the pseudo Saffman lift force becomes significant there due to a drastic increase in $|\partial u_{f0}/\partial y|$. This lift force is pointed towards the pipe center, thus generating a particle-free zone near the wall.

For small particles, e.g. the particles of 100 and 200 μm dia in Lee & Durst's (1982) experiments, the transverse particle transport in the central portion of the pipe becomes significant where the fluid drifting velocity is no longer negligible, due to the significant values of $\tilde{\eta}_e$ and

$$\left|\frac{1}{\alpha}\frac{\partial \alpha}{\partial y}\right|$$

there. In the near-wall region, the fluid drifting velocity is increased due to an increase in

$$\left|\frac{1}{\alpha}\frac{\partial \alpha}{\partial y}\right|$$

with $\tilde{\eta}_e$ remaining significant. The pseudo Stokes drag force based on this fluid drifting velocity is greatly increased due to a significant increase in the value of $\tilde{a}$ associated with a sizeable peaking of the apparent turbulent viscosity of the fluid in a region close to the wall, as shown by the correlation of Lee & Börner (1987). There, the particles are thrown towards the wall. Once they have passed through the transverse location where the time-mean longitudinal fluid and particle velocities, u_{f0} and $\bar{u}_p$ respectively, match each other, instantaneously the particles will lead the fluid in longitudinal motion. Then the pseudo Saffman lift force, now much enhanced due to the significant increase in the value of $\tilde{h}$ associated with a sizeable peaking of the apparent turbulent viscosity of the fluid there and reversed in direction due to a change of sign of the slip velocity in the longitudinal direction $(\bar{u}_p - u_{f0})$, starts to help propel the particles towards the wall. Eventually, the particles will, on collision with the wall, bounce back and during this short excursion in the close near-wall region, the particles will continue to move faster than the fluid in the longitudinal direction.

Using the present particle transport theory with the use of the apparent turbulent viscosity of the fluid, which is felt by a particle in the suspension, provided by the correlation of Lee (1987), and the turbulent flow properties of air in a two-phase suspension flow, provided by the correlations of Lee & Börner (1987), Lee & Wong (1987) carried out numerical calculations of particle motion in an upward turbulent glass particle–air two-phase suspension flow in a vertical pipe. Their results are found to agree closely with measurements by laser–Doppler anemometry by Lee & Durst (1982); in particular, in the aforementioned peculiar particle migratory behavior according to particle size in the near-wall region.

CONCLUSION

A new unified theory has been formulated for the study of motion of particles in a turbulent dilute particle–fluid two-phase suspension flow. The required inputs on the turbulence structure of the fluid phase in the two-phase flow are provided by realistic correlations from experimental measurements over a wide range of physical and flow parameters.

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